INFLUENCE OF VISCOSITY AND HEAT CONDUCTIVITY ON THE PROPAGATION OF SOUND IMPULSES IN AN INHOMOGENEOUS MOVING MEDIUM

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Approximate equations of short waves propagating in a moving, inhomogeneous, viscous and heat conducting medium are derived. These equations include the principal nonlinear and dissipative terms, and make possible the description of the two-dimensional structure within the wave. Two limiting cases of the problem on propagation of sound impulses are considered. The exact particular solution is obtained for the case when the influence of the dissipative terms is vanishingly small, i.e. when the medium can be assumed perfect, and this solution in turn yields the laws of decay of weak shock waves in a moving inhomogenous nonviscous gas with zero heat conductivity. These laws were investigated before in [1-3], but they were obtained for the first time as the result of a straightforward solution of the first approximation equations describing the gas flow within the wave. Asymptotic form of the sound impulses and the laws of decay over very long periods of time are principally governed by the dissipative terms, and nonlinear terms are not essential at this stage.

1. We consider a problem concerning the propagation of sound waves in a moving, inhomogenous, viscous and heat conducting medium. Let t denote time; x, y, z are Cartesian coordinates; $\mathbf{v}$ is mass velocity vector with components v_x, v_y, v_z; g_x, g_y, g_z are components of the vector of acceleration due to gravity; ρ is density; p is pressure; a is adiabatic velocity of sound; λ_1, λ_2 are the first and second viscosity coefficients; $\varkappa$ is the heat conductivity coefficient; γ is the ratio of specific heats at constant pressure c_p and constant volume c_v, and α is the thermal expansion coefficient. Then we can write the following closed system of equations describing the flow of an arbitrary two-parameter medium in the form [4-6]

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_j}{\partial x_j} = 0 \tag{1.1}$$

$$\rho\left(\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k}\right) = -\frac{\partial p}{\partial x_i} + \rho g_i + \frac{\partial}{\partial x_k}\left[\lambda_1\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)\right] + \frac{\partial}{\partial x_i}\left[\left(\lambda_2 - \frac{2}{3}\lambda_1\right)\frac{\partial v_j}{\partial x_j}\right] \tag{1.2}$$

$$\frac{\partial p}{\partial t} - a^2\frac{\partial \rho}{\partial t} + v_k\left(\frac{\partial p}{\partial x_k} - a^2\frac{\partial \rho}{\partial x_k}\right) = \frac{a^2\alpha}{c_p}\frac{\partial}{\partial x_k}\left[\frac{\varkappa}{\rho\alpha}\left(\frac{\gamma}{a^2}\frac{\partial p}{\partial x_k} - \frac{\partial \rho}{\partial x_k}\right)\right] +$$
$$+ \frac{\lambda_1}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)^2 + \left(\lambda_2 - \frac{2}{3}\lambda_1\right)\left(\frac{\partial v_j}{\partial x_j}\right)^2 \tag{1.3}$$

The indices i, j, k assume the values 1, 2 and 3 and, as usual, repeated indices denote summation. Temperature T and specific entropy s can, therefore, be found from [5,6]

$$\rho a^2\alpha\, T ds = c_p(dp - a^2 d\rho), \qquad \rho a^2\alpha dT = \gamma dp - a^2 d\rho \tag{1.4}$$

Let now a wave propagate through an unperturbed medium whose parameters v_{0i}, p_0, $\rho_0, \ldots$ are known functions of coordinates, and let the differences between the values of these parameters within the wave denoted by $v_{0i}, p_0, \rho_0, \ldots$ and the unperturbed values be small. We can then assume, as in [7], that the unperturbed flow of the medium can be described in terms of the equations for a perfect gas flow, resulting from (1.1) - (1.3) when $\lambda_1 = \lambda_2 = \varkappa = 0$.

$$\frac{\partial \rho_0 v_{0j}}{\partial x_j} = 0, \quad \rho_0 v_{0k} \frac{\partial v_{0i}}{\partial x_k} = -\frac{\partial p_0}{\partial x_i} + \rho_0 g_i, \quad v_{0k}\left(\frac{\partial p_0}{\partial x_k} - a_0^2 \frac{\partial \rho_0}{\partial x_k}\right) = 0 \tag{1.5}$$

In the acoustic approximation, the velocity of the sound wave relative to the gas molecules will be equal to the unperturbed velocity of sound, therefore the wave in question can be identified with the C_+-characteristic $\varphi\ (t,\ x_k) = 0$ of the equations of motion of a perfect gas, which can be obtained within the same approximation from [8]

$$\frac{\partial \varphi}{\partial t} + v_{0j} \frac{\partial \varphi}{\partial x_j} + a_0 \left[\left(\frac{\partial \varphi}{\partial x_j}\right)^2\right]^{1/2} = 0 \tag{1.6}$$

Characteristic curves of (1.6) which coincide, in our approximation, with the bicharacteristics of the Euler equations, can be regarded as trajectories of the elements of the wavefront surface or as rays [8]. They are given by the following ordinary differential equations:

$$\frac{dx_i}{dt} = v_{0i} + a_0 n_i, \quad \frac{dn_i}{dt} = (n_i n_j - \delta_{ij})\left(\frac{\partial a_0}{\partial x_j} + n_k \frac{\partial v_{0k}}{\partial x_j}\right) \tag{1.7}$$

Here n_i denote the components of the unit vector $\mathbf{n}$ normal to the characteristic surface and δ_{ij} are Kronecker deltas. Since v_{0i}, a_0 , as well as their derivatives are known, the solutions of (1.6) and (1.7) can be obtained in advance and written as

$$\varphi\ (t,\ x_k) = 0, \quad x_i^\circ = x_i^\circ(t,\ x_{0k}), \quad n_i^\circ = n_i^\circ(t,\ x_{0k}) \tag{1.8}$$

where x_{0k} denote the coordinates of an initial point defining a certain ray. This ray intersects the characteristic surface at the point (x_k°) and the superscript $^\circ$ will denote the quantities taken at this point and depending only on time. Coordinates of (x_k°) are given by (1.8).

Let us introduce a moving Cartesian coordinate system $x_i'\ (x_1, x_2, x_3)$ associated with the wave. Formulas for x_k° in (1.8) will define the new origin. Since the basic changes in the wave take place in the direction normal to the wavefront, we shall direct the x_1-axis along the vector $\mathbf{n}$ normal to the characteristic. Components n_i° are given by the last formula of (1.8). Two other axes, x_2, x_3 of the new coordinate system lie in the plane tangential to the characteristic. The actual choice of these axes is not essential, but it can be shown that the transformation to new coordinates is simpler, when the components of two mutually perpendicular unit vectors $\mathbf{e}_2$ and $\mathbf{e}_3$ satisfy, in the tangent plane, the following differential equations:

$$\frac{de_{2i}^\circ}{dt} = n_i^\circ\left(\frac{\partial a_0^\circ}{\partial x_j} + n_k^\circ \frac{\partial v_{0k}^\circ}{\partial x_j}\right) e_{2j}^\circ, \quad \frac{de_{3i}^\circ}{dt} = n_i^\circ\left(\frac{\partial a_0^\circ}{\partial x_j} + n_k^\circ \frac{\partial v_{0k}^\circ}{\partial x_j}\right) e_{3j}^\circ$$

whose solutions

$$e_{2i}^\circ = e_{2i}^\circ\ (t,\ x_{0k}), \quad e_{3i}^\circ = e_{3i}^\circ\ (t,\ x_{0k}) \tag{1.9}$$

can also be assumed known in advance.

We note that the equations for dn_i°/dt, de_{2i}°/dt, de_{3i}°/dt are not independent, consequently an additional condition of uniqueness of the corresponding coordinate vector is required for the closure of each system.

With (1.8) and (1.9) known, we can establish the transformation formulas relating the fixed and moving coordinate systems

$$t = t', \quad x_i = x_i^\circ(t) + e_{ji}^\circ(t)\, x_j' \qquad (e_{1i}^\circ \equiv n_i^\circ) \qquad (1.10)$$
$$t' = t, \quad x_i' = e_{ij}^\circ(t)\,[x_j - x_j^\circ(t)]$$

We shall assume that the characteristic surface $\varphi(t, x_k) = 0$ moving through the physical space, passes through each of its points once only. Then the formulas (1.8) and (1.9) define three vectors $\mathbf{e}_1^\circ, \mathbf{e}_2^\circ, \mathbf{e}_3^\circ$ at any point at that instant at which the characteristic passes through it. Let us assign the above triplet of vectors to this point, i.e. let us assume that three mutually perpendicular unit vectors $\mathbf{e}_1 \equiv \mathbf{n}, \mathbf{e}_2 \equiv \boldsymbol{\tau}_2, \mathbf{e}_3 \equiv \equiv \boldsymbol{\tau}_3$ are given at each point and that their components $e_{ij}(x_k)$ depend only on the coordinates of this point. Projections of the vector $\mathbf{v}'$ at this point on the directions defined above will be denoted by $v_n', v_{\tau 2}', v_{\tau 3}'$, respectively, and they will be related to the components of $\mathbf{v}'$ in the moving coordinate system by the formulas

$$v_i' = n_i v_n' + \tau_{2i} v_{\tau 2}' + \tau_{3i} v_{\tau 3}' \qquad (1.11)$$

where $n_i, \tau_{2i}, \tau_{3i}$ denote the components of $\mathbf{n}, \boldsymbol{\tau}_2, \boldsymbol{\tau}_3$ in the new coordinate system.

Let us now assume that the gas flow in question represents a wave which is not only weak, but also short, i.e. that the dimensions of the region in which the perturbations are concentrated and, particularly, the characteristic wavelength Λ (in the $\mathbf{n}$-th direction) are small, compared with the radius of curvature of its front and with the characteristic size of the inhomogeneity of the medium. We shall denote the smaller of the above two quantities by L, and the characteristic size of the wave in the plane tangent to its wavefront by Δ. The excess values of all the parameters vary more slowly in this plane, and there are no preferred directions. We shall also denote the typical relative values of the longitudinal and transverse components v_n' and $v_{\tau 2}'$, $v_{\tau 3}'$ of the perturbed velocity vector by ε and ω respectively, assuming at the same time that the density, pressure and sound velocity perturbations have the same order of smallness ε as v_n'.

Let us convert to dimensionless variables in the new coordinate system

$$x_1 = L\Lambda x_1^\times, \quad x_2 = L\Delta x_2^\times, \quad x_3 = L\Delta x_3^\times, \quad t' = (L / a_{00})\, \theta t^\times$$
$$v_{0i} = a_{00} v_{0i}^\times, \quad a_0 = a_{00} a_0^\times, \quad p_0 = \rho_{00} a_{00}^2 p_0^\times, \quad \rho_{00} = \rho_{00}\rho_0^\times \qquad (1.12)$$
$$v_n' = a_{00}\varepsilon v_n'^\times, \quad v_{\tau 2}' = a_{00}\omega v_{\tau 2}'^\times, \quad v_{\tau 3}' = a_{00}\omega v_{\tau 3}'^\times$$
$$a' = a_{00}\varepsilon a'^\times, \quad p' = \rho_{00} a_{00}^2 \varepsilon p'^\times, \quad \rho' = \rho_{00}\varepsilon \rho'^\times$$

Here a_{00} and ρ_{00} are constant (along the ray) values of the velocity of sound and density at the origin (x_{0k}) of the ray, all dimensionless quantities accompanied by the multiplication cross superscript are of the order of unity and $\Lambda, \varepsilon, \omega$ are small compared with unity.

Few more remarks are convenient at this point before passing to the process of transforming the initial equations.

We find that in the short waves the velocity component and the derivatives of all the flow parameters in the direction normal to the surface of the wavefront exceed in magnitude the corresponding quantities perpendicular to the normal. Thus we can assume that [2,3]

$$\omega \ll \varepsilon, \qquad \Lambda \ll \Delta \qquad (1.13)$$

The assumption that $\Lambda \ll \theta$ eliminates the usual case of linearization of the Navier-Stokes equations.

We also introduce the Reynolds, Péclet and Prandtl numbers

$$N_{\mathrm{Re}} = \frac{\rho_{00} a_{00} L}{{}^4/_3 \lambda_1 + \lambda_2}, \quad N_{\mathrm{Pe}} = \frac{\rho_{00} a_{00} c_p L}{\varkappa}, \quad N_{\mathrm{Pr}} = \frac{N_{\mathrm{Pe}}}{N_{\mathrm{Re}}} \tag{1.14}$$

Viscosity and heat conductivity coefficients are, as usual, of the same order of magnitude, and sufficiently small for [4,5]

$$1/N_{\mathrm{Re}} \ll \Lambda, \quad N_{\mathrm{Pr}} \sim 1 \tag{1.15}$$

to hold.

The quantities λ_1, λ_2, $\varkappa$, and c_p, α, γ are known functions of state and can be assumed given outside the wave. Their perturbations can be neglected in the first approximation since, by (1.15), their inclusion complicates only slightly the computations without affecting the final approximate equations [5].

By (1.12), all dimensionless values of the parameters of the unperturbed motion of the medium and of their derivatives are of order unity. Compared with them, their perturbations are small, and their maximum order is ε in the adopted notation. Since the dimension of the perturbed region is also small, the gradients in the perturbed zone (maximum order ε/Λ) are comparable with the gradients of the initial distributions. But in this case the curvatures of the perturbation profiles (maximum order ε/Λ^2) should be even more significant compared with the curvatures of the distributions of the unperturbed values of the parameters. This therefore leads to yet another inequality

$$1 \ll \varepsilon / \Lambda^2 \tag{1.16}$$

Finally, we note that although the matrix of the components n_i, τ_{2i}, τ_{3i}, is nearly diagonal within the wave, i.e. at small (compared with L) distances from the new coordinate origin, the final approximate equations may include the derivatives of the components whose values are nearly zero and these derivatives may, in general, be of order unity. In addition, we must distinguish between the quantities with and without the superscript $^\circ$, since the only purpose of the former is to define the new coordinate system, while the latter may undergo variation within this system. Although these quantities differ little from each other in the perturbed zone, nevertheless the final equations include the term arising from these differences. In [7], which deals with weak shock waves generated by an airplane moving through an inhomogeneous atmosphere with dissipation, neglect of the above circumstance is reflected in the approximate equation obtained. A similar remark is made in [10].

Let us insert (1.12) recalling (1.11), into the perturbation equations obtained from (1.1) - (1.3) after eliminating from the latter the terms corresponding to an unperturbed flow (1.5) and converting to the moving coordinate system by means of (1.10). We shall use the relations (1.11) and (1.13) - (1.16), retaining the major terms only in the resulting expressions. Omitting the complicated derivation, we shall give the resulting approximate equations for the short waves (neglecting the multiplication cross superscript)

$$\rho' = (\rho_0^\circ / a_0^\circ)\, v_n', \quad p' = \rho_0^\circ a_0^\circ v_n'$$

$$\frac{\omega}{\Lambda}\frac{\partial v_{\tau 2}'}{\partial x_1} = \frac{\varepsilon}{\Delta}\frac{\partial v_n'}{\partial x_2}, \quad \frac{\omega}{\Lambda}\frac{\partial v_{\tau 3}'}{\partial x_1} = \frac{\varepsilon}{\Delta}\frac{\partial v_n'}{\partial x_3} \tag{1.17}$$

$$\frac{1}{\theta}\frac{\partial v_n'}{\partial t} + \frac{\varepsilon}{\Lambda} m_0^\circ v_n' \frac{\partial v_n'}{\partial x_1} + x_1 \frac{d \ln u_{0n}^\circ}{dt}\frac{\partial v_n'}{\partial x_1} + \frac{\omega}{\varepsilon \Delta}\frac{a_0^\circ}{2}\left(\frac{\partial v_{\tau 2}'}{\partial x_2} + \frac{\partial v_{\tau 3}'}{\partial x_3}\right) +$$

$$+ \frac{d \ln M^\circ}{dt} v_n' = \frac{1}{\Lambda^2 N_{\mathrm{Re}}}\frac{1}{2\rho_0^\circ}\left(1 + \frac{\gamma - 1}{N_{\mathrm{Pr}}}\right)\frac{\partial^2 v_n'}{\partial x_1^2}$$

where

$$M^\circ = \exp\left\{\frac{1}{2}\int\left[\frac{d\ln(\rho_0{}^\circ a_0{}^\circ)}{dt} + a_0{}^\circ\frac{\partial n_j{}^\circ}{\partial x_j} + (2m_0{}^\circ - 1)\frac{\partial v_{0j}{}^\circ}{\partial x_j} + \frac{\partial v_{0n}{}^\circ}{\partial x_1}\right]dt\right\} = u_{0n}\left(\frac{\rho_0{}^\circ f^\circ}{a_0{}^\circ}\right)^{1/2}$$

$$v_{0n} = v_{0j}n_j, \quad u_{0n} = a_0 + v_{0n}, \quad m = {}^1/_2\,\rho^{-3}a^{-2}\,[\partial^2 p/\partial(1/\rho)^2]_s$$

The expression for M° in which the integral was taken along the ray, was integrated in [11]. The quantity u_{0n} represents the projection of the vector of the so-called ray velocity $\mathbf{u}_0 = a_0\mathbf{n} + \mathbf{v}_0$ with which the wavefront surface propagates through the medium on the normal to this surface and f denotes the area of the wavefront surface element contained within the elementary ray tube (i.e. a tube of small cross section generated by the rays). We note that all the quantities accompanied by the superscript $^\circ$ depend only on time, and shall omit this superscript as well.

The first two formulas of (1.17) follow from the equations of continuity and from the projection of the Navier-Stokes equation on the x_1-axis. Their significance is that in our approximation the gas is compressed adiabatically and that the Riemann relation characterizing a plane running sound pulse through the medium [4] holds. The next two equations of (1.17) obtained from the projections of the Navier-Stokes equations on the x_2- and x_3-axes with the condition $\omega/\Lambda = \varepsilon/\Delta$ characterizing structurally inhomogeneous flows indicate the absence of vortices in the perturbed zone. Thus, simplifying continuity equations (1.1) and Navier-Stokes equations (1.2), we also obtain expressions characterizing motions in perfect media. The dissipation terms appear in the last equation of (1.17), which follows from the mass transport equation (1.3) with allowance for (1.1), (1.2) and from the first two equations of (1.17).

In the most general case $\theta \sim 1$, $\varepsilon \sim \Lambda$, $\omega \sim \Lambda^{3/2}$, $\Delta \sim \Lambda^{1/2}$, $1/N_{\mathrm{Re}} \sim \Lambda^2$, the equations defining the short waves become

$$\frac{\partial v_{\tau 2}'}{\partial x_1} = \frac{\partial v_n'}{\partial x_2}, \qquad \frac{\partial v_{\tau 3}'}{\partial x_1} = \frac{\partial v_n}{\partial x_3} \tag{1.18}$$

$$\frac{\partial v_n'}{\partial t} + m_0 v_n'\frac{\partial v_n'}{\partial x_1} + x_1\frac{d\ln u_{0n}}{dt}\frac{\partial v_n'}{\partial x_1} + \frac{a_0}{2}\left(\frac{\partial v_{\tau 2}'}{\partial x_2} + \frac{\partial v_{\tau 3}'}{\partial x_3}\right) + \frac{\ln M}{\partial t}v_n' = \frac{1}{2\rho_0}\left(1 + \frac{\gamma - 1}{N_{\mathrm{Pr}}}\right)\frac{\partial^2 v_n'}{\partial x_1^2}$$

Equations most closely resembling (1.18) are given in [7]. However, the motion within the wave was assumed to be immediately quasionedimensional; certain other differences already noted are also involved.

2. We shall now consider short waves with quasionedimensional structure and the corresponding condition $\omega \ll \varepsilon\Delta$. From the last equation of (1.17) we have

$$\frac{1}{\theta}\frac{\partial v'}{\partial t} + \frac{\varepsilon}{\Lambda}m_0 v'\frac{\partial v'}{\partial x_1} + x_1\frac{d\ln u_{0n}}{dt}\frac{\partial v'}{\partial x_1} + \frac{d\ln M}{dt}v' = \frac{1}{\Lambda^2 N_{\mathrm{Re}}}\left(1 + \frac{\gamma - 1}{N_{\mathrm{Pr}}}\right)\frac{\partial^2 v'}{\partial x_1^2} \tag{2.1}$$

where both principal nonlinear and dissipative terms are retained, and v_n' is replaced by v'.

First let us assume that the dissipative factors in (2.1) can be neglected, i.e. that $1/N_{\mathrm{Re}} \ll \Lambda^2$. In this case (2.1) yields the following equation describing the gas flow in a weak wave propagating in a moving inhomogenous perfect medium

$$\frac{\partial v'}{\partial t} + m_0 v'\frac{\partial v'}{\partial x_1} + x_1\frac{d\ln u_{0n}}{dt}\frac{\partial v'}{\partial x_1} + \frac{d\ln M}{dt}v' = 0 \quad (\theta \sim 1,\ \varepsilon \sim \Lambda) \tag{2.2}$$

If we now omit the second and third terms containing the derivative $\partial v' / \partial x_1$ from the left-hand side of this equation, the equation of geometrical acoustics for the wavefronts will result. This is easily integrated yielding the well known law of variation of sound-wave amplitude [4,11,13]

$$v' = \frac{f(x_1)}{u_{0n}} \left(\frac{a_0}{\rho_0 f}\right)^{1/2} \tag{2.3}$$

where the function $f(x_1)$ defines the wave profile which does not vary this approximation so that its choice is arbitrary. All the same, if we wish to follow the behavior of both, the wavefront and the wave profile, then retention of the linear term $x_1 (d \ln u_{0n} / dt) (\partial v' / \partial x_1)$ is advisable even in the acoustic approximation. This term appears as a result of the fact that a wave propagates in an inhomogeneous moving atmosphere with varying velocity, and this results in "linear" distortion of its profile.

It is clear that the acoustic solution will not yield the asymptotic laws of decay of perturbations as $t \to \infty$, even for a perfect medium. This is obtained simply by substituting (2.3) into (2.2) with t tending to infinity.

The asymptotic laws of decay of sound pulses in an inhomogeneous moving medium can be obtained from (2.2). As $t \to \infty$, the profiles of weak shock waves tend to a linear form [4] and the gas flow behind such waves can be described by the following solution of (2.2):

$$v' = \frac{x_1}{M u_{0n}} \left(c_1 + \int_{t_1} \frac{m_0}{M u_{0n}} \, dt\right)^{-1} \tag{2.4}$$

where c_1 is an arbitrary constant, the integration is carried out along a ray and t_0 coincides with the origin of this ray.

Let us now obtain the law of variation of the intensity v_*' of the shock wave along the ray. Both the intensity and the wavelength λ_* depend on time only, and the dependence follows from (2.4).

The velocity N of propagation of a weak shock wave to within the first order of smallness is given in the fixed coordinate system by [3,4]

$$N = u_{0n} + \frac{\lambda_*}{u_{0n}} \frac{du_{0n}}{dt} + \frac{1}{2} m_0 v_*'$$

Taking into account conversion formulas (1.10) together with (1.8) and (1.9), we have

$$\frac{d\lambda_*}{dt} = \frac{\lambda_*}{u_{0n}} \frac{du_{0n}}{dt} + \frac{1}{2} m_0 v_*' \tag{2.5}$$

Differentiating (2.4) between the quantities v_*' and λ_* along the trajectory of the shock wavefront and use of (2.5) we obtain the following expression for v_*':

$$\frac{1}{v_*'} \frac{dv_*'}{dt} + \frac{1}{2} \frac{m_0}{M u_{0n}} \left(c_1 + \int_{t_*}^{t} \frac{m_0}{M u_{0n}} \, dt\right)^{-1} + \frac{1}{M} \frac{dM}{dt} = 0 \tag{2.6}$$

which on integration yields

$$v_*' = \frac{c_2}{u_{0n}}\left(\frac{a_0}{\rho_0 f}\right)^{1/2}\left[c_1 + \int_{t_0}^{t} \frac{m_0}{u_{0n}^2}\left(\frac{a_0}{\rho_0 f}\right)^{1/2} dt\right]^{-1/2}, \qquad \lambda_* = c_2 u_{0n}\left[c_1 + \int_{t_0}^{t} \frac{m_0}{u_{0n}^2}\left(\frac{a_0}{\rho_0 f}\right)^{1/2} dt\right]^{1/2} \tag{2.7}$$

where c_2 is an arbitrary constant.

If the amplitude v_0' and the wavelength λ_0 of the shock wave at $t = t_0$, are both known, then we can find c_1 and c_2 by substituting these values into (2.7) and setting $t = t_0$.

Formulas (2.7) express the well known [1-3,11] laws of decay of weak shock waves in a moving inhomogenous medium, but were arrived at in the present paper by obtaining the exact solution of the approximate equations derived above. As before, they remain valid as long as $\lambda_* \ll L$.

In the second limiting case [4,6] when the time intervals become very long ($\theta \sim 1$, $\varepsilon \ll \Lambda$, $1 / N_{Re} \sim \Lambda^2$) , dissipative factors begin to play a decisive role. Nonlinear term in (2.1) can be neglected, yielding

$$\frac{\partial v'}{\partial t} + x_1 \frac{d \ln u_{0n}}{dt}\frac{\partial v'}{\partial x_1} + \frac{d \ln M}{dt} v' = \frac{1}{2\rho_0}\left(1 + \frac{\gamma - 1}{N_{Pr}}\right)\frac{\partial^2 v'}{\partial x_1^2}$$

The substitution of variables

$$w = Mv' = \sqrt{\rho_0 f / a_0}\, u_{0n} v', \qquad \tau = {}^1/_2 \int_{t_0}^{t} \rho_0^{-1}[1 + (\gamma - 1) / N_{Pr}]\, dt$$

makes it possible to rewrite it in a simpler form,

$$\frac{\partial w}{\partial \tau} + x_1 \frac{d \ln u_{0n}}{d\tau}\frac{\partial w}{\partial x_1} = \frac{\partial^2 w}{\partial x_1^2} \tag{2.8}$$

Thus, at the final stage, the decay of sound pulses is described by a parabolic equation of the type (2.8) and will, therefore, become more pronounced than in the case of a perfect medium. We note that the quantity $w = \sqrt{\rho_0 f / a_0}\, u_{0n} v'$ which appears in (2.8) is constant in the geometrical acoustics approximation; this follows from (2.3).

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THREE-DIMENSIONAL RUNNING WAVES IN A BAROTROPIC GAS

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The equations of potential triple waves in a barotropic gas with an arbitrary equation of state are obtained. The properties of the solutions for contiguous flows of the double- and triple-wave type are investigated. The solutions of certain three- dimensional self-similar problems of three pistons are solved in the case of a "heavy" gas with a high initial velocity of sound. These problems concern three planes forming an infinite trihedral angle within which the gas is at rest at the instant $t = 0$, whereupon the planes begin to retract from the gas at high constant velocities.

1. A system of equations of triple waves for a polytropic gas in the hodograph space of the velocities u_1, u_2, u_3 was derived in [1]. Double waves in a barotropic gas for nonsteady potential two-dimensional flows were considered in [2] (see also Suchkov, Applying the method of differential constraints to gas dynamics problems. Candidate's thesis, Siberian Branch of the Academy of Sciences USSR, Novosibirsk). Some of the results of [2] constitute minor generalizations of the results obtained in [3,4] for a polytropic gas.

The equations of potential unsteady third-rank waves [1] for a gas with the equation of state $p = f(\rho)$ (p is the pressure, ρ is the density) can be derived exactly as for a polytropic gas. Proceeding as in [1], we introduce as our unknown functions the enthalpy

$$H(u_1, u_2, u_3) = \int \frac{dp}{\rho}$$

and the "deployment" function

$$\Pi(u_1, u_2, u_3) = \sum_{k=1}^{3} x_k u_k - \varphi - tH - \frac{t+1}{2}\,(u_1^2 + u_2^2 + u_3^2) \qquad (1.1)$$

Here x_k are Cartesian coordinates and φ is the velocity potential. We obtain the following system of equations for these functions H and Π :